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# APPLICATION OF MATHEMATICAL VOCABULARY

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "Application of Mathematical Vocabulary" submitted by Michael Makar, in partial fulfillment of the requirements for the Degree of Master of Education.

## ABSTRACT

The purpose of this study was to investigate the relationship between the vocabulary aspect of reading and the problem-solving aspect of mathematics. More precisely, the study was designed to determine the degree of relationship between knowledge of mathematical vocabulary and success in application of concepts that involve this vocabulary, to problem solving in geometry at the grade ten level.

Eighty-three students were selected for the study. This was done by choosing one class from each of three high schools in the Edmonton Public School System. Three tests were administered to the subjects. The first was a Mathematical Vocabulary Test on which the subjects were required to write definitions for items of mathematical vocabulary. The second was a Concept Test which required the subjects to solve problems requiring no computation, by the selection of the correct answer from among four possible choices. The third was a Problem-Solving Test which required computation in order to arrive at the correct answer which was also to be selected from among four possible choices. All of the testing was carried out during the month of June in 1977.

The data was analyzed by the computation of correlation coefficients between the Mathematical Vocabulary Test and the Concept Test and also between the Mathematical

Vocabulary Test and the Problem-Solving Test. In addition to this the mean, variance and standard deviation were computed for the results of each of the three tests.

Significant correlational relationships were found between the students' knowledge of mathematical vocabulary and their ability to deal with concepts and solve problems which involved this vocabulary. In addition to this the subjects found the task of writing formal definitions to items of mathematical vocabulary significantly more difficult than dealing with concepts and solving problems which involved the items of mathematical vocabulary.

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# CHAPTER I

## INTRODUCTION

An examination of the curriculum outlines for the province of Alberta obviates the conclusion that reading in the content fields is one of the most neglected aspects of public school education in this province. This can probably be attributed to the fact that traditionally it was assumed that if a student was taught to read (i.e. narrative material) there would be more or less automatic carry-over from this general reading instruction to reading in the content fields. The assumption resulted in the teaching of reading during one specific period and content field material during other specific periods. Thus, little or no consideration was given to the specialized reading skills required within each content field. Gradually it was realized that too much carry-over was assumed, and that general reading skills were not automatically applied successfully to reading content area materials.

Despite the recognition that reading programs, of necessity, must teach the students to read in the content areas, instruction is still to a great extent concentrated on narrative (basal) type of reading material. This, according to Jackson (1968), can be largely attributed to the

fact that the reading program is limited by time, and as such,teaching for the transfer of training to reading other types of material remains as the basis of instruction in reading.

## PROBLEM

In the content area of mathematics, one of the recent trends at all grade levels of public school education has been the shift in emphasis from the teaching of routine skills to the teaching of basic mathematical ideas that structure the discipline. This has placed increased emphasis on teaching the concepts that are integral to the understanding of the discipline. In turn, the increased intellectual rigour required by this approach has placed greater demands upon the reading ability of the students.

Research such as that of Burks and Bruce (1955) and Jan-Tausch (1960) has indicated that the understanding a reader obtains from print is partially a function of the number and clarity of concepts the reader has fixed through words. Since mathematics has its own special vocabulary, according to Aaron (1965), Bell (1965), Bond (1966), Clark (1969), Eagle (1948), Foran (1933), Johnson (1944), Johnson (1949), Johnson (1952), Lyda and Duncan (1967), Morgenstern (1969), Muelder (1969), Rudman (1952), Spencer (1960), Spitzer (1952) and Weber (1963), it would appear that

students must be familiar with this vocabulary to read and successfully interpret concepts and problems in which this vocabulary is employed. This presents the problem of determining the degree of relationship between students' knowledge of specialized mathematical vocabulary and their success in the solving of mathematical problems which involve concepts generated by this vocabulary.

## PURPOSE OF THE STUDY

The purpose of this study is to investigate the degree of relationship between the knowledge of specific mathematical vocabulary of grade ten students and their success in solving problems which employ this mathematical vocabulary. The mathematical vocabulary and related problems were derived from the "Mathematics Ten" geometry program authorized for use in the Edmonton Public Schools during the 1976-77 school year.

## DEFINITION OF TERMS

Non-specialized vocabulary: A vocabulary which consists of those words known to the majority of members of a cultural group as a result of a commonality of experience.

Specialized vocabulary: A vocabulary which contains those words known to a group within a culture as a result of a

specialized experience. These words are used for the purpose of communication within a particular field.

Mathematical vocabulary (terminology): A specialized vocabulary which contains those words which are specific to the field of mathematics.

Concepts: Ideas or generalizations drawn from particular instances; that is, specific information organized in some fashion in order to identify relationships among them.

Problems: The applications of concepts, which involve knowledge of mathematical vocabulary, to situations that require some manipulation of numerical data.

## HYPOTHESES

From the investigation of research studies and in view of what the investigator proposes to do in this study, the following null hypotheses have been formulated:

I. There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Concept Test (i.e. a multiple-choice problem solving test which does not require any computation in arriving at the correct alternative).

II. There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Problem-Solving Test (i.e. a mathematical multiple-choice problem-solving test which requires computation in order to arrive at the correct alternative).

## ADDITIONAL QUESTION

Will the students' scores be consistently lower, at the same level, or consistently higher on the Mathematical Vocabulary Test than on either the Concept Test or the Problem-Solving Test?

## LIMITATIONS OF THE STUDY

The generalizability of the findings of this study are limited in accordance with the following consideration:

> The task of writing formal definitions for words (i.e. mathematical terms) represents only one aspect of vocabulary knowledge; mainly that of formally expressing the understanding of the words. This may be limited by the student's power of expression and as a result the student will be unable to demonstrate his/her understanding of the word.

## ASSUMPTIONS

1. It is assumed that each student's performance on the tests used in this study is indicative of his ability on those aspects that these tests purport to measure.
2. It is assumed that the method of selection of the sample will provide randomness adequate for the purpose of this

study.

3. It is assumed that the readability of the tests used in this study will not be a determining factor and thus lies outside of the limits of this study.

## SIGNIFICANCE OF THE STUDY

It is hoped that the results of this study will indicate whether or not students' knowledge of the mathematical vocabulary has a significant relationship to success in problem solving in mathematics. A significant degree of relationship would indicate the necessity for a good formal knowledge of the mathematical vocabulary in order to enable students to better cope with the reading involved in problem solving. This would provide support for the formal teaching of vocabulary skills along with the mathematical vocabulary within the settings of mathematics instruction rather than simply an introduction of the terminology which appears to be the current practice.

## OVERVIEW OF THE INVESTIGATION

In Chapter I the problem was identified and the study was outlined. The results of previous studies and literature related to the problem will be presented in Chapter II. Chapter III will describe the sample, the testing instruments and procedures, the method of qualitative

analysis and the statistical procedures. The results of the statistical analysis of the data and its interpretation will be presented in Chapter IV. Chapter V will contain the conclusions based on the findings and the implications for further research.

# CHAPTER II

## REVIEW OF THE LITERATURE AND RESEARCH

The review of the literature and research is organized into two sections. The first section deals with the nature of reading in Mathematics and the second section deals with the review of selected research with respect to the relationship between reading ability and mathematical achievement.

### THE NATURE OF READING IN MATHEMATICS

According to Corle (1972) research has shown that knowledge of mathematical concepts alone will not guarantee success in mathematics, particularly in the area of written, or verbal mathematical problems. He contends that the ability to arrive at correct solutions to these problems is directly related to the ability to read and interpret them. Weintraub (1967) points out that in order to read skillfully in any content area two characteristics are required of the reader. First, he must have a general reading ability and secondly, he must have specific reading skills required by the discipline of the content field. Pressey and Pressey (1921) and Briggs (1966) indicate that skill in silent

reading depends, to a great extent, upon the nature of the material to be read. Thus, a good reader in one content area may very likely be a poor reader in another content area. In light of this it becomes necessary to examine the reading skills required for successful reading of mathematics.

Although research is not conclusive there appears to be general agreement between mathematics educators and reading educators on the skills of reading mathematics that are common to other content areas, as well. These common skills are listed below with the references cited:

1) Noting details: (Bond and Wagner, 1966; Chase, 1960; Fay, 1965; Muelder, 1969; Rudman, 1952; Smith, 1963; Spencer and Russell, 1960; Spitzer 1952).
2) Following directions: (Bond and Wagner, 1966; Fay, 1965; Morgenstern, 1969; Rudman, 1952; Smith, 1963; Spitzer, 1952).
3) Organizing and relating facts: (Bond and Wagner, 1966; Fay, 1965; Niles, 1969; Rudman, 1952; Smith, 1963; Spencer and Russell, 1960; Spitzer, 1952; Weber, 1963).
4) Judging the relevancy of information: (Bond and Wagner, 1966; Fay, 1965; Rudman, 1952; Spitzer, 1952; Weber, 1963).

5) Recalling important facts: (Morgenstern, 1969; Muelder, 1969; Smith, 1963).

6) Locating information: (Morgenstern, 1969; Niles, 1969; Smith, 1963; Spencer and Russell, 1960).

7) Reading graphic materials: (Morgenstern, 1969; Niles, 1969; Rudman, 1952, Strain, 1969).

8) Forming visual impressions: (Bond and Wagner, 1966; Weber, 1963).

In addition to the reading skills which are operative in the discipline of mathematics and common to other content areas, specific reading skills are required. In the literature reviewed there is a general concensus of opinion between reading educators and mathematics educators regarding what these specific reading skills unique to mathematics entail. These skills are briefly discussed below with references cited.

1) Adjustment to Vocabulary: (Aaron, 1965; Bell, 1965; Bond and Wagner, 1966; Clark, 1969; Eagle, 1948; Fay, 1950; Fay, 1965; Foran, 1933, Johnson, 1944; Johnson, 1949; Johnson, 1952; Lessenger, 1925; Lyda and Duncan, 1967; Morgenstern, 1969; Muelder, 1969; Rudman, 1952; Smith, 1963; Spencer and Russell, 1960; Spitzer, 1952; Weber, 1963).

Before a student can read mathematics, he must learn

the language of this discipline. A very careful definition of the mathematical terms is necessary since it is often individual words which give meaning to what is being read. Thus, understanding every word becomes critical when that single word may influence the reading of an entire passage, example, or problem. Mathematics has a highly technical vocabulary characterized by a high degree of precision in meaning. Such terms as perimeter, denominator, quotient, numerator, subtrahend, exponent, radius, diameter, and triangle serve as examples of the many words that are rarely met outside of mathematics. In many cases, these words are abstract and backgrounds of experience in relation to the real world have to be carefully laid so that a student does not merely verbalize, but understands as well.

Also part of the highly technical vocabulary of mathematics are common words and phrases that have special meanings in mathematics different from the common meanings the student knows. Meanings such as _mixed_ in "mixed numbers," _times_ in "five times seven," _carry_ in "multiplication," _improper_ in "improper fraction," _root_ and _plane_ are all examples of such specialized interpretations.

In order to interpret the shorthand of mathematics, the symbols used must be recognized and must have meaning. That is, the student must be able to read a new symbolic system, different from the symbolic system of words. There

are symbols for numbers: 1, 35, 240; for geometric figures: $\angle ABC$, $\overrightarrow{CD}$; for operations: +, -, X, ÷; for relations: =, <, >; and highly specialized uses of common symbols such as parentheses and brackets. To add to the difficulty, the symbols and their meanings are abstract, that is, are considered apart from any application to a particular object.

Not only does the student need to know the symbols, he also needs to know the pattern which governs their arrangement. For example, to read a numeral, each digit must be observed and read according to its potential value. This value is derived from the plan of the decimal system based on grouping by tens and assigning a value (place value) to each position. This is basically a simple scheme which when applied repeatedly provides numerals for any number considered. However, there is not always a familiar pattern to follow. For example, ten gives no cue relationship to either nine or eleven; twelve gives no clue as to the nature of the next name, thirteen. The three in thirteen and the five in fifteen have strange names. The decades--twenty, thirty, forty, etc.--do not follow the names originally assigned to two, three, or four. It is not until sixty that the numerals are named by the names originally assigned.

Another form of mathematical shorthand is the use of abbreviations, particularly with measures. Again, there appears to be no pattern to follow, as witness lb. for pound,

cwt. for hundred weight and oz. for ounce. Yet, these symbols must be learned in order to read mathematics.

Because of the hierarchical structure of mathematics, it is not surprising that knowledge of symbols is rapidly extended to knowledge of combinations of symbols. Those combinations which use only symbols to convey complete ideas are commonly called mathematical sentences, and represent a new form of sentence structure for the student. These generally begin as equations: $4 + 5 = 9$; then a variable is introduced: $4 + n = 9$; and a new use for a common symbol, a letter, must be learned. Another combination of symbols; one step further on the hierarchy, is the algorithm: the form used in writing the symbols for computation purposes. Reading these computational procedures is a very specialized type of reading in which the student not only must know the symbols but must know the basis for each step in the computation.

2) Adjusted Rate of Reading: (Aaron, 1965; Bell, 1965; Bond and Wagner, 1966; Clark, 1969; Morgenstern, 1969; Muelder, 1969; Rudman, 1952; Spencer and Russell, 1960; Spitzer, 1952; Tinker and McCullough, 1968).

Mathematics must be read slowly, deliberately, carefully, and with intense concentration. The number of pages of mathematics that a student reads each day is small when

compared to the number of pages he reads in other content areas, or for pleasure. This is due to the nature of the mathematical material. It is concise, contains more ideas per line and page than most other writing, and is written at a highly abstract level. That is, it is concerned with ideas and symbols rather than with actual objects. Re-reading and vertical reading are often required, and particular attention must be paid to individual words. Thus, the student must make a definite adjustment in his rate and style of reading.

3) Reading Charts, Graphs and Tables: (Aaron, 1965; Bond and Wagner, 1966; Fay, 1950; Morgenstern, 1969; Rudman, 1952; Spencer and Russell, 1960).

Although suggested by many writers as a special skill in reading mathematics, this might be interpreted to mean that it is a reading skill to be taught in mathematics classes because of the quantitative nature of the data involved. Certainly, the ability to get information from these sources is widely used in other content areas. In mathematics, the skill of reading charts, graphs, and tables is extended to include their construction as well.

4) Reading in Verbal Problem Solving: (Bell, 1965; Bond and Wagner, 1966; Clark, 1969; Fay, 1965; Morgenstern, 1969; Muelder, 1969; Riedesel, 1969; Rudman, 1952; Spencer and Russell, 1960; Spitzer,

1952; Weber, 1963).

Problems are often considered the reading material of mathematics. This is not the case as explanations and examples also require reading. However, it is in the problem-solving phase of mathematics that reading skills have their major application, because this is considered the central or crucial area of mathematics. The skills of reading most often related to problem solving are the literal, interpretative, and critical comprehension skills. Because the position of solving problems is uppermost on the hierarchy of mathematical skills, all the specialized reading skills discussed in the above sections apply to problem solving.

In addition to the specific reading skills required by the reader of mathematics the nature of the material itself must be considered. Pressey and Pressey (1921) and Briggs (1966) indicate the skill in silent reading depends, to a great extent, upon the nature of the material to be read.

According to Bell (1965), Bond and Wagner (1966), Clark (1969), Heddons and Smith (1964), Hill (1967), Kerfoot (1961), Morgenstern (1969), Muelder (1969), Reed (1968), Spencer and Russell (1960), Spitzer (1952), Stauffer (1966) and Weber (1963) mathematics materials requiring reading are unique in that they involve content which is markedly

different from any other material the student must read. The vocabulary, both words and symbols, is unique as has been pointed out previously. The typical textbook material lacks continuity, is very terse and concise, has little contextual relationship and mixes technical vocabulary and vernacular vocabulary with the symbols of mathematics. Thus, in reading such materials many of the habits or techniques that are suitable for reading narrative materials must be extensively revised and new skills specific to the content area must be added. In addition to this, the readability of materials written for specific grade level is often above the reading level of the students for which it has been prescribed, and as such adds to the difficulty in the reading of mathematics materials.

## RESEARCH ON READING IN MATHEMATICS

It is easy to understand how reading ability could effect performance in mathematics; especially in the area of problem solving. Monroe (1918) found that the same problem could be verbally stated twenty-eight different ways in arithmetic textbooks. This finding, coupled with Linville's (1969) conclusion that both vocabulary level and syntactic structure determine the difficulty of verbal problems makes obvious the fact that reading ability is involved in problem solving in mathematics.

Research on the relationships of reading to mathematics may be classified into four categories which overlap to some degree. These categories can be described as (1) general reading ability and mathematics, (2) specific reading skills and mathematics, (3) readability of mathematics materials, and (4) vocabulary and mathematics. Selected studies in each of these categories are reviewed in the sections that follow.

## General Reading Ability and Mathematics

Studies into the relationship of reading to mathematics date back into the 1920's and 1930's. Stevens (1932) concluded that ability in fundamental operations of mathematics was more closely correlated with ability in problem solving than with general reading ability. This finding was confirmed by Morton (1953) who reported that skill in problem solving correlated highly with skill in fundamental operations and intelligence, but showed a low, though positive, correlation with general reading speed. Other studies in this area, however, do not necessarily agree with this conclusion.

Balow (1964) used sixth grade students in an effort to determine whether or not general reading ability was significantly associated with problem-solving ability and if the level of computational skills was significantly

associated with problem-solving ability. He found that general reading ability did have an effect on problem-solving ability and that computational ability had a significant effect on problem-solving ability. He noted that this differed from most of the previous studies in this area and suggested that his use of pupils with a total range of reading ability rather than good and poor readers separately might be the accounting factor. He concluded that it was important to consider children's reading ability when teaching problem-solving.

Chase (1960) conducted a study involving sixth grade pupils in an effort to determine which skills and intellectual factors were related to the ability to solve verbal problems in arithmetic. He obtained a correlation of r = .40 between the "Gates Reading Survey - Form 2" and the "Iowa Every Pupil Test-Arithmetic" (problem section) and a correlation of r = .34 between the "Primary Mental Abilities Verbal" and the "Iowa Every Pupil Test-Arithmetic" (problem section) tests. He concluded that the extent to which a pupil notices specific items of information when he reads could serve as a prediction of success in problem solving.

To investigate arithmetic problem-solving as it is influenced by reading comprehension, computation, abstract verbal reasoning, and arithmetic concepts, Martin (1964) tested grade four and grade eight students. She obtained

correlations of .61, .64, .66, and .60, respectively, between problem solving and abstract verbal reasoning, reading, arithmetic concepts, and computation. The partial correlation between problem solving and reading with computation held constant was .52 and between problem solving and computation with reading held constant was .43. Martin concluded that skills in reading were necessary in problem solving in addition to the basic facility of arithmetic computation.

Pitts (1952) in a study involving negro high-school girls obtained a correlation of $r = .53$ between The Daw Test of Functional Competence in Mathematics and the Iowa Silent Reading Tests (reading grade level). Pitts concluded that there is a marked positive relationship between functional competence in mathematics and reading grade level.

Skillman (1972) in an attempt to explore the effectiveness of teaching a college algebra class by a method that stressed instruction in reading, compared statistically the mathematical achievement of the students receiving the reading emphasis with the achievement of students not receiving the reading emphasis. He found the difference between the mean gain scores, significant at the .05 level, in favor of the college algebra class receiving the reading treatment.

Smith (1976) used third grade non-readers (i.e. students reading one grade level below the expected grade level in reading) in an effort to determine the degree of dependence of mathematics achievement on reading achievement. She concluded that students who do not achieve success in reading can learn mathematics at a rate at least equal to that of an average student.

Despite the lack of total agreement among the studies in the area there seems to be some indication of positive correlation between general reading ability and mathematics; particularly in the problem-solving area. However, the degree of relationship does not appear to be particularly high, as all of the results necessarily depend upon the reading and mathematics tests used. General reading scores very often include tests of paragraph reading, reading of literary (narrative) materials for main ideas, and general vocabulary items while mathematics scores may include tests of competence in computation which involve very little reading.

## Specific Reading Skills and Mathematics

There seems to be general agreement among researchers and educators that certain specific reading skills are necessary for success in mathematics. Lessenger (1925) found that specific instruction in reading the signs

of operation had favorable effects on mathematics computation scores. In a study involving general language ability, vocabulary and specific reading skills, Hansen (1944) found significant differences between good and poor problem solvers.

Treacy (1944) studied the relationship of reading skills to the ability to solve problems in arithmetic. His sample consisted of seventh grade pupils in two junior high schools. He administered two problem solving tests, the Otis Quick-Scoring Mental Test and 15 tests of various kinds of reading ability. He hypothesized that the difference in means in the various reading skills of good and poor problem solvers would be zero. The hypothesis was rejected and he concluded that the ability to solve arithmetic problems is a result of a composite of specific reading skills, rather than general reading ability.

Fay (1950) used sixth graders in a study in an effort to determine the relationship between specific reading skills and achievement in mathematics. He made the comparison on the upper and lower ninety pupils in his sample group with respect to reading achievement. He found that superior readers did not achieve any better on the arithmetic than inferior readers, and concluded that arithmetic achievement was not specifically related to any group of reading abilities. His findings appear to be in

conflict with those of Treacy (1944) as well as the majority of others which have been carried out in this area.

Corle (1958) asked grade six students, in individual sessions, to solve eight arithmetic problems. He evaluated the students' performance for accuracy, concept of meaning of the problems, reasoning used in the solution of the problems, confidence of the pupils in their answers to the problems, understanding of selected vocabulary and oral reading of the problems. He computed contingency coefficients for each of the latter five factors using chi square. He found that all coefficients except the oral reading were significantly different from zero at the .01 level and that the oral reading was significantly different from zero at the .06 level. He obtained a contingency coefficient of .82 between the pupils' concept of what the problems meant and their accuracy in solving the problem.

Alexander (1960) used seventh grade students as subjects in an effort to determine whether there was any relationship between problem solving ability and a number of selected factors. He found a significant relationship between problem-solving ability and the following factors: (1) mental age, (2) ability to understand verbal concepts, (3) general intelligence, (4) reading comprehension, (5) reading vocabulary, (6) arithmetic computation, (7) arithmetic concepts, (8) ability to analyze verbal

problems, (9) general ability to interpret data, (10) perception of relationship involving comparison of data, and (11) recognition of limitations of given data.

Cleland and Toussaint (1962) used intermediate grade pupils in a study to determine the relationship of reading and listening to arithmetic computation. They obtained the following correlations significant at the .01 level: (1) r = .49 between the Gates Reading Survey - Form 2 and the American School Arithmetic tests, (2) r = .46 between the Durrell-Sullivan and the American Arithmetic tests, and (3) r = .41 between the Step Listening Comprehension and the American School Arithmetic tests.

Muscio (1962) used grade six students in a study of factors related to quantitative understanding. Correlations between quantitative understanding and arithmetic reasoning, arithmetic computation and mathematical vocabulary ranged from .73 to .81. Correlations between quantitative understanding and various reading tests correlated between .55 and .78. Intercorrelations among the factors of quantitative understanding, arithmetic computation, mathematical vocabulary, and I.Q. ranged from .71 to .81.

Call and Wiggin (1966) in an experiment, with high school students, investigated the effects of two different methods of teaching second-year algebra. The experimental group was taught by Wiggin, who was an English teacher with some training in the teaching of reading but no experience

in teaching mathematics. The control group was taught by Call who was an experienced mathematics teacher. The major difference between the two instructional methods was that Wiggin stressed understanding the meaning of words in mathematics problems and translating the verbal statements into mathematical symbols where possible. This procedure was more like that used in the teaching of reading rather than in mathematics. The result of this experiment was that the experimental group did better on the criterion test in mathematics than the control group. This result remained true even when the initial difference on the reading and mathematics test scores were statistically partialled out.

Cotrell (1967) conducted a study with the purpose of focusing on certain language factors which appeared to be associated with underachievement in arithmetic of third grade students. He found significant correlations between arithmetic vocabulary and arithmetic concepts ($r = .54$ at the .05 level), independent reading and arithmetic concepts ($r = .59$ at the .05 level), instructional reading and arithmetic concepts ($r = .54$ at the .05 level), paragraph meaning and arithmetic concepts ($r = .86$ at the .01 level), and language achievement and arithmetic concepts ($r = .47$ at the .05 level). He interpreted the positive correlations which he observed among reading, general mental ability and arithmetic factors as being due to a general

linguistic ability rather than the pivotal variable of general intelligence which appears to be a determining factor in both reading and mathematical ability.

The transfer effects of reading remediation to arithmetic computation were studied by Gilmary (1967). In this study the experimental group was provided with remedial measures in both reading and arithmetic while the control group received instruction in arithmetic only. The experimental group showed an average gain, during a six-week period of approximately one-third of a grade more than the control group. This difference was found to be significant at the .01 level. Gilmary concluded that reading skills stressed in arithmetic had significant transfer value for arithmetic classes.

The research in this area seems to be conclusive in its findings and suggests a need for direct teaching of specific reading skills and abilities applicable to mathematics.

## Readability of Mathematics Materials

The nature of mathematical material and the relationship of reading ability to success in mathematics suggest the importance of careful selection of mathematical materials which students must read. Since the textbook has always been an important source of mathematical material

for teachers of mathematics and often the only source provided, its readability warrants examination.

Johnson (1952) discovered that a program of word enrichment is needed for the understanding of textbooks. She found that hundreds of words in mathematics texts were unknown to the fifth grade pupils tested. This led her to conclude that word enrichment is needed in order to help students deal with the vocabulary in their textbooks and lesson materials.

Repp (1960) tabulated 3,329 words extracted from five widely used third-grade level textbooks and found that 1,379 to 2,096 of these words were new to third graders. In a similar study Kerfoot (1961) examined the vocabulary in six arithmetic textbook series for grades one and two. He compiled a list of 49 basic words for grade one and 370 words for grade two. Of the grade two words 62 did not appear on either the Gates List of Vocabulary for Primary grades or the Dale List of 769 Easy Words. He concluded that the list which he had prepared represented a selection of words which a child is likely to meet in the first or second year of mathematics instruction and as such this is where they should be taught.

Kolson (1963) in a study of quantitative vocabulary concluded that of 229 arithmetic words used by the kindergarteners in the study 70% were quantitative words. In a

similar study, Stauffer (1966) compared the vocabularies in primary grade basal readers and textbooks of three content areas, one of which was mathematics. He found very little overlap of vocabularies between the basal reader and the content area textbooks as well as a lack of uniform usage in the content area texts. He recommended a program of word attack skills emphasizing meaning in each of the content areas. Like Kolson and Stauffer, Reed (1968) also analyzed the vocabulary of prescribed mathematics textbooks for grades one through three. She found no significant agreement between the vocabularies in the mathematics textbooks and the vocabulary in the reading series used by the same pupils. She also found a greater, but not significant agreement between the mathematics vocabularies studied and the standard word list.

The studies reviewed above provide considerable evidence of the existence of a disproportionate number of unfamiliar words in mathematics textbooks used by school children. Given this situation it becomes evident that some measure of textbook readability is necessary. The most common and popular readability measures which have been used are the Dale-Chall formula, the Spache formula and the Cloze technique. These concern the relationship of vocabulary and syntax to ease of reading.

Heddens and Smith (1964) used the Dale-Chall and

Spache formulas to study five commercially available series of mathematics textbooks prepared for grades one through six. The Spache formula was applied to the textbooks used in grades one through three and the Dale-Chall formula was applied to the textbooks in grades four through six. They concluded that the readability of all of the textbooks was generally above the assigned grade level. In addition to this they found considerable variation within each textbook as well as among the texts of each series studied.

Kane (1968) contends that the readability formulae used on ordinary English prose are usually inappropriate for use on mathematical English, and gives detailed reasons to support his contention. According to Kane ordinary English prose and mathematical English differ in that: (1) letter, word, and syntactical redundancies are different, (2) in contrast to ordinary English where there might be several denotations, in mathematical English the names of mathematical objects usually have only one denotation, (3) adjectives are more important in mathematical English than in ordinary English, and (4) the grammar and syntax of mathematical English are less flexible than in ordinary English.

In spite of Kane's (1968) claim that there is no logically defensible approach to assessing the readability of mathematics materials, a study by Hater (1969) of the

Cloze technique suggests that this procedure can be quite useful. In Hater's analysis the Close tests were found to be highly reliable and valid predictors of the comprehensibility of mathematical English passages designed for grades seven through twelve.

Regardless of the readability measure that might be employed for the assessment of the readability of mathematical materials the research reviewed in this area establishes the fact that the vocabulary of mathematical materials is frequently at a higher level than the performance level of the students where the materials are used. In addition to this, mathematical vocabulary does not greatly overlap that of the materials used in the teaching of reading.

## Vocabulary and Mathematics

Among the reviewed research of the relationships between mathematical abilities and specific aspects of reading ability a good deal of attention has been directed toward vocabulary. The majority of these studies appear to be concerned with relating vocabulary knowledge to success in some aspect of mathematics. For example, Foran (1933) found that technical terms and other unfamiliar words interfered greatly with performance in problem solving, at different age and grade levels.

A study by Buckingham (1937) of the relationship between vocabulary and ability in first year algebra revealed a significant relationship between the knowledge of words and the ability to solve algebra problems. In addition to this, the definitions produced by students revealed three levels of development: (1) completely undeveloped, (2) concretely developed but abstractly not completely developed, as in the case where an example or illustration is given in place of a definition, and (3) completely developed abstractly with concomitant concrete illustration, as in the case where the definition is accompanied by a graphical illustration.

In a study involving seventh-grade students, Johnson (1944) sought to determine whether improvement in knowledge of specific mathematical vocabulary led to an improvement in the solution of problems which involved the use of the specific mathematical terms. For both the experimental and control groups the study was divided into three periods, with each period being preceded and followed by tests in vocabulary and problem solving based upon what had been taught during the respective periods. A minimum emphasis was placed upon the computation involved in the problem. Significant gains in favor of the experimental group were found on each of the vocabulary post-tests, with the greatest gains appearing during the first and second

periods. Significant gains were also found in favor of the experimental group in the area of problem solving. Not only did the experimental group achieve greater gains in both vocabulary and problem solving, but the superiority was maintained for pupils of practically all levels Of mental ability.

Eagle (1948) studied the relationship between various reading abilities and success in mathematics with two groups of grade nine algebra students. Employing a multiple-choice type of exam, he found that general vocabulary correlated with success in mathematics only .25 and with algebra .31, while mathematical vocabulary correlated .53 with general mathematics and .48 with algebra. He concluded that reading comprehension and mathematics success were largely related to mental age, however, reading comprehension was still significant in relation to success in mathematics when mental age was held constant.

Johnson (1949) studied the relation of specific reading skills to problem solving using eighth grade students in Chicago. He administered six tests of arithmetic problems and six tests of Primary Mental Abilities to the students. He obtained the following correlations between the Primary Mental Ability Vocabulary and the standardized achievement tests in Arithmetic: Stanford Arithmetic Reasoning (.51), Chicago Survey Test in Arithmetic (.50),

and the Stone Reasoning Test (.45). Furthermore, he found that the Primary Mental Ability Vocabulary test correlated more highly with scores on a non-standardized test composed of problems with numbers (.40) than with scores on a test of problems without numbers (.26). He concluded that it might be correct to say that problem solving in arithmetic is related to general intelligence through factors of vocabulary and reasoning.

Vanderlinde (1964) experimented with nine fifth-grade classes matched with nine control classes on I.Q. and scores on achievement tests in vocabulary, reading comprehension, arithmetic concepts and arithmetic problem solving. The experimental classes studied a different list of eight quantitative terms each week for twenty to twenty-four weeks, after which the achievement tests were administered. He found that the experimental group achieved significantly higher scores on a test of arithmetic problem solving than the control group. He stated that the inability of pupils to comprehend the statement of the problem and lack of vocabulary knowledge were among significant reasons why they have difficulty in the solving of arithmetic problems.

A study by Lyda and Duncan (1967) was designed to find the effect of direct study of quantitative vocabulary on problem solving with second-grade pupils. Terms were

extracted from the second-grade arithmetic textbook regularly and studied by the pupils. All of the terms used were considered to have at least one quantitative meaning with the exception of the cardinal and ordinal numbers. After studying the selected terms for a part of the regular arithmetic period for eight weeks, the pupils showed gains, significant at the .01 level, in the areas of reading, arithmetic computation and arithmetic reasoning. They concluded that a significant growth in problem solving ability resulted from this direct study of quantitative vocabulary.

Linville (1969) used fourth-grade students in a study designed to investigate whether the degree of syntax used in sentences which state verbal problems and/or the level of vocabulary used in the statement of the verbal problems are factors which contribute significantly to the degree of difficulty of the problems when the computational operations are held constant. Four arithmetic word-problems tests, each consisting of the same problems but varying in difficulty of syntax and vocabulary were administered: (1) easy syntax, easy vocabulary, (2) easy syntax, difficult vocabulary, (3) difficult syntax, easy vocabulary, (4) difficult syntax, difficult vocabulary. The results revealed significant main effects in favor of both the easy syntax and easy vocabulary tests. He concluded that both syntactic structure and vocabulary level,

with vocabulary level perhaps more crucial, are important factors in the ability to solve verbal arithmetic problems.

The research reviewed in the area of vocabulary indicates a consistently positive and strong connection between mathematics, especially problem solving and mathematical vocabulary. This is not surprising since vocabulary knowledge is basic to comprehension and students must read and comprehend the problems before they can apply the mathematical concepts necessary for the solutions.

## SUMMARY

From the literature and research reviewed in this chapter it can be concluded that:

1) Reading in mathematics is different from reading narrative material in that it is highly specialized requiring, in addition to general reading ability, the development of specific reading skills. These involve adjustment to the vocabulary and symbolism unique to mathematics and a slower, more careful and deliberate rate of reading which requires intense concentration.

2) The relationship between general reading ability and success in learning mathematics is positive but not consistently significant to any degree.

3) The general relationship between specific reading skills and success in learning mathematics, particularly

in the area of problem solving, is positive and consistently significant to a high degree.

4) The readability of mathematical materials is an important consideration for the learning of mathematics, particularly in the area of mathematical vocabulary.

5) There is a consistently positive and significantly strong connection between success in learning mathematics and knowledge of mathematical vocabulary, suggesting that general vocabulary, quantitative vocabulary and the vocabulary of symbolism in mathematics should receive careful consideration by teachers of mathematics.

CHAPTER III

# EXPERIMENTAL DESIGN

In this chapter, the experimental design will be described. Information regarding the description of the population and sample, and an outline of the procedures to investigate the hypothesis will be presented. In addition to this, the construction, administration and scoring of the test instruments will be discussed and the analysis of the data will be outlined.

## POPULATION AND SAMPLE

The population of this study consisted of grade ten students enrolled in "Mathematics Ten" in three Edmonton Public Schools during the year 1976-77. Grade ten students were selected for two reasons. First, the researcher wished to examine the relationship between the vocabulary aspect of reading and the problem-solving aspect of mathematics at the high school level. Secondly students are introduced to deductive geometry at the grade ten level and shown how this discipline is developed from the definition of the mathematical vocabulary it employs to the development of the concepts it involves. Since the course is introduced

and developed from basic fundamentals and is not dependent on former knowledge in the area of geometry, it provided a good setting to study the relationship between mathematical vocabulary knowledge and success in problem solving which required understanding of concepts that involved the mathematical vocabulary. That is to say the inability to recall concepts formerly learned would not become an interfering factor that might present itself at more advanced grade levels. Aside from this, no other special characteristic with respect to the population or sample were considered.

The sample consisted of eighty-three students from the three high schools in the same system. One class was selected from each of the high schools on the basis of the teacher involved. The teachers were chosen by the investigator because of their willingness to cooperate in the study as well as their similar styles of teaching which was determined by the investigator in an interview with each teacher. The provision for a similar treatment of material was felt to be necessary by the investigator in order to control the variation in teaching techniques which was not an aspect of this study.

# TESTING PROCEDURES

During the month of June, 1977, at the completion of the geometry section of the "Mathematics Ten" course, three forty-minute (i.e. the length of a class period in high school) examinations were administered to each of the three classes involved. Since these were written examinations and no special instructions were required, the administration was carried out by the regular teacher of each class. The subjects had been informed approximately two weeks in advance to provide adequate preparation time. This was done in order to provide for a testing situation that would parallel the regular classroom testing situation. Such an action was deemed appropriate since this was to be a "field" study. According to Kerlinger (1965) a field study is a scientific inquiry aimed at the discovery of relationships and interactions among educational variables in their real social and structural settings. Since data collection did not require any manipulation of the variables involved, the field study seemed to provide the most appropriate vehicle for the collection of the required data.

The tests were administered in three successive forty-minute periods. The first was a written examination dealing with the Mathematical Vocabulary, the second was a Concept Test which required no computation, and the third was a Problem-Solving Test which required computation.

## CONSTRUCTION OF THE TESTS

In order to measure the variables involved in this study, three separate tests were constructed by the investigator. The variables to be measured were the students' knowledge of mathematical vocabulary (see Appendix A), the students' ability to comprehend the mathematical concepts represented by the vocabulary selected for this study (see Appendix B), and the students' achievement in solving mathematical problems by comprehending the meaning of the selected vocabulary before applying computational skills (see Appendix C). The tests constructed were a Mathematical Vocabulary Test, a Concept Test, and a Problem-Solving Test.

### (i) The Mathematical Vocabulary Test

The Mathematical Vocabulary Test was designed to measure the students' knowledge of the mathematical vocabulary as defined in this study. It consisted of ten items of mathematical vocabulary extracted from the current text book (Wilcox, 1968), used in the teaching of geometry in the mathematics ten program. These were terms like "reflex angle" and "perpendicular bisector" which are specific to the field of mathematics. (See Appendix A for the entire test). In the selection of the terms for the Mathematical Vocabulary Test the following procedure was used: The

topics outlined for the geometry ten program were reviewed and all of the items of mathematical vocabulary were listed. The terms on the list which were frequently used in the development of concepts and in problem-solving situations were marked. The context in which each of these terms appeared in the text book was examined to ensure usage strictly as a mathematical term. This was done in order to eliminate any confusion of usage which might arise from the selection of terms like "root" and "plane" each of which has a specific meaning in the field of mathematics and other meanings outside of the field of mathematics. Ten terms were selected from the list which was drawn up from the corpus of the entire course. This number was chosen in order to allow all students to complete the test in a period of forty minutes which is the length of a class period in high school. The vocabulary items were selected by the investigator on the basis of specificity to mathematics and frequency of usage in the development of concepts and in problem solving. They were approved on the same basis by the mathematics teachers involved in the teaching of the classes selected for the study.

(ii) The Concept Test

The Concept Test was designed to measure the students' ability to apply concepts that involved vocabulary from the Mathematical Vocabulary Test. Unlike the

Mathematical Vocabulary Test which required the production of definitions on the part of the students, this test required the students to display an understanding of concepts by recognition of the solutions. That is to say the students did not have to write or do any computation in order to arrive at the correct alternative but rather select the proper one from among four possible answers. For example, the item on the Concept Test (see Appendix B for the entire test) which involved the item "complementary angles" from the Mathematical Vocabulary Test read:

> In triangle PQR, angles P and Q are complementary. The triangle must be
>
> A. equilateral
> B. isosceles
> C. obtused-angled
> D. right-angled

As in the case of the Mathematical Vocabulary Test the topics for the mathematics ten geometry program were reviewed and possible items for the Concept Test were selected from a bank of items used on tests in the program in previous years by the investigator. From among the items on the list of possible questions ten were selected by the investigator on the basis that each item applied one of the terms from the Mathematical Vocabulary Test and each was judged by the investigator to be of average level of difficulty. The items were approved by the mathematics teachers, involved in the teaching of the classes selected

for this study, on the basis of the appropriateness of each item in respect to the vocabulary item which was incorporated into the item. In addition to this the teachers looked at the appropriateness of each item, on the Concept Test, in terms of the material covered in their respective classes to see that the concepts involved had received adequate coverage.

(iii) The Problem-Solving Test

The Problem-Solving Test was designed to measure the students' ability to comprehend the meaning of the selected vocabulary before applying computational skills to solve the problem. Each item on the Problem-Solving Test applied an item of vocabulary from the Mathematical Vocabulary Test as in the case of the Concept Test. Unlike the Concept Test which required only selection of the correct alternative, the Problem-Solving Test required the students to display understanding of the concepts involved by the application of numerical computations in order to arrive at the solution. For example, the item on the Problem-Solving Test (see Appendix C for the entire test) which involved "complementary angles" from the Mathematical Vocabulary Test read:

If an angle is nine times its complement, the angle is

A. 8 degrees
B. 81 degrees
C. 72 degrees
D. 9 degrees

In order to solve the problem the student would have to understand the meaning of the term "complementary angles" and then might set up and solve the equation $x = 9(90 - x)$ or use logic to deduce the correct answer from among the four alternatives.

As in the case of the Mathematical Vocabulary Test and the Concept Test the topics for the mathematics ten geometry program were reviewed and problems for the Problem-Solving Test were selected from a bank of items used on tests in the program in previous years by the investigator. From among the items on the list of possible problems ten were selected by the investigator on the basis that each item applied one of the terms from the Mathematical Vocabulary Test and was judged to be of average level of difficulty. The items were approved by the mathematics teachers, involved in the teaching of the classes selected for this study, on the basis of appropriateness of each item in respect to the mathematical term which was incorporated into the item and adequacy of coverage of the underlying concepts and computational skills.

SCORING OF THE TESTS

The definitions on the specialized mathematical vocabulary test were scored on the basis of right or wrong. The definition was considered correct if it complied with the last of the three methods of definition outlined in Gerstein's (1949) classification system. The first is the descriptive or concretistic method wherein the term is defined as an object at a concrete or sensory level (e.g. median of a triangle - is a line figure with a beginning and an end point). The second is the functional or usage method wherein the term is defined in terms of function or application (e.g. median of a triangle - is a line segment used to divide a triangle into two parts). The third is the conceptual or categorical method wherein the term is defined on the basis of a general class and the distinctive features (e.g. median of a triangle - is the line segment joining any vertex of a triangle to the midpoint of the opposite side).

The decision to employ Gerstein's third method of definition as a basis for the scoring of the Mathematical Vocabulary Test was arrived at by mutual agreement of the investigator and the two independent scorers. This was done following the examination and discussion of Gerstein's three methods of definition in respect to the characteristics exhibited by mathematical vocabulary. The

characteristics in question are mathematical vocabulary's precision of meaning and in many instances its abstract nature. The precision of meaning arises from the fact that it is often individual words which give meaning to what is being read, and as such, precise meaning becomes critical since a single word can influence the comprehension of a concept or a problem. In addition to this the meanings of terms like "deductive reasoning" or "alternate angles" are to a high degree abstract in nature and as such cannot be defined precisely under Gerstein's first method of definition (i.e. the descriptive or concretistic) or his second method of definition (i.e. the functional or usage).

The reliability of scoring the Mathematical Vocabulary Test was established through inter-scorer agreement. Prior to the scoring of the vocabulary test papers the investigator and the two independent scorers established guidelines with respect to what would constitute a correct definition for each of the ten mathematical terms. The guidelines were based on Gerstein's conceptual or categorical method. Under the method a term is defined on the basis of a general class and distinctive features. As an example, the term "median of a triangle" to be marked correct would have to say that it is a line segment (the general class) and that it joins a vertex of a triangle to the mid-point of the opposite side (the distinctive

features). Upon agreement on the guidelines for each of the ten terms on the test, the investigator and the independent scorers each independently scored all of the items on the test papers. The three scorers' results for each subject were copied onto a master sheet and then punched onto a computer card. A separate computer card was punched for each subject. The cards were then fed into the computer (DERS) which produced the following correlations matrix.

| | Column 1 | Column 2 | Column 3 |
|---|---|---|---|
| Row 1 | 1.000 | 0.957 | 0.975 |
| Row 2 | 0.957 | 1.000 | 0.930 |
| Row 3 | 0.975 | 0.930 | 1.000 |

According to Guilford (1965) reliability correlation coefficients are expected in the range of .70 to .98, however, they should be judged in respect to the circumstances under which they are obtained and as such interpreted in the light of these circumstances. Since the criterion for correctness of definitions was restricted to Gerstein's highest method of definition (i.e. the conceptual or categorical method) in this study, it was hoped that the inter-scorer reliability correlation coefficients would exceed .90. Since the correlation coefficients obtained in the matrix exceed .90 the reliability of scoring the mathematical vocabulary test was considered as being satisfactory.

The Concept Test and the Problem-Solving Test were

both of the multiple-choice type, and as such, their format provided for objective scoring methods. Since tests of this type can be machine scored the computer at the University of Alberta (DERS) was employed for this purpose.

TREATMENT OF THE DATA

Each of the three test scores for each subject was copied onto a master sheet and then punched out on a computer card. A separate computer card was punched out for each subject and contained the following information:

1) the individual's coded identification number,
2) the individual's score on the Mathematical Vocabulary Test,
3) the individual's score on the Concept Test
4) the individual's score on the Problem-Solving Test.

The data were analyzed as follows: Mean, variance, and standard deviation were computed for each of the three tests administered in this study. These statistics were computed in order to compare the students' performance on the Mathematical Vocabulary Test to their performance on the Concept Test and their performance on the Problem-Solving Test. This was done in order to see if the

students would find the production of definitions more difficult, of the same difficulty, or less difficult than dealing with questions on concepts or solving problems where a choice of answers is provided. In addition to the analysis of each test on the basis of mean, variance, and standard deviation, correlation coefficients were calculated for the pairs of tests involved in this study. Correlation coefficients were computed between the Mathematical Vocabulary Test and the Concept Test, and the Mathematical Vocabulary Test and the Problem-Solving Test. This was done in order to determine the degree of relationship between the students' comprehension of mathematical vocabulary and their ability, on the one hand, to comprehend the mathematical concepts that employed this vocabulary and, on the other hand, their ability to understand and solve problems which employed this vocabulary. In order to determine whether the correlation coefficients were significant, the "t" test was employed. This test is applied by assuming the null hypothesis (i.e. that the correlation coefficient is zero) and then computing the "t" value for the correlation coefficient involved.

## CHAPTER IV

## ANALYSIS OF THE DATA

This chapter will present the findings of this study. The analysis of the results will be presented in two main sections. The first will present the findings with respect to the hypotheses of this study. This will be done in terms of statistical analysis of the correlation coefficients obtained between the results of the tests used for the analysis of the hypotheses. The second section will present the findings with respect to the additional question of this study. This will be done in terms of statistical analysis of the mean, variance, and standard deviation of each of the three tests used in this study.

### FINDINGS WITH RESPECT TO THE HYPOTHESES

#### Hypothesis One

There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Concept Test (i.e. a mathematical multiple-choice problem-solving test which does not require any computation in arriving at the correct alternative).

This hypothesis was analyzed by means of calculating

the correlation coefficient between the results on the Mathematical Vocabulary Test and the results of the Concept Test. The correlation coefficient obtained between the pair of tests was r = .632 as can be seen in Table I which shows the correlation coefficients matrix obtained from the analysis of the data. This correlation coefficient was found to be significant at the .001 level as can be seen in Table II which shows the "t" values obtained from the analysis of the data. Thus null hypothesis one was rejected.

Hypothesis Two

There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Problem-Solving Test (i.e. a mathematical multiple-choice problem-solving test which requires computation in order to arrive at the correct alternative).

This hypothesis was analyzed by means of calculating the correlation coefficient between the results on the Mathematical Vocabulary Test and the results of the Problem-Solving Test. The correlation coefficient was r = .559 as shown in Table I. This correlation coefficient was also found to be significant at the .001 level as shown in Table II. Thus null hypothesis two was also rejected.

TABLE I

MATRIX OF CORRELATION COEFFICIENTS FOR PAIRED SETS OF SCORES OBTAINED FROM THE ADMINISTRATION OF THE TESTS USED IN THIS STUDY

| | Math Vocab. Test | Math Concept Test | Math Problem-Solving Test |
|---|---|---|---|
| Math Vocab. Test | 1.000 | 0.632 | 0.559 |
| Math Concept Test | 0.632 | 1.000 | 0.459 |
| Math Problem-Solving Test | 0.559 | 0.459 | 1.000 |

TABLE II

MATRIX OF "t" VALUES TO TEST THE SIGNIFICANCE OF THE OBTAINED CORRELATION COEFFICIENTS

| | Math Vocab. Test | Math Concept Test | Math Problem-Solving Test |
|---|---|---|---|
| Math Vocab. Test | -- | 7.336* | 6.063* |
| Math Concept Test | 7.336* | -- | 4.648* |
| Math Problem-Solving Test | 6.063* | 4.648* | -- |

* Significant at the .001 level

# FINDINGS WITH RESPECT TO THE ADDITIONAL QUESTION

## Additional Question

Will the students' scores be consistently lower, at the same level or consistently higher on the Mathematical Vocabulary Test than on the Concept Test or the Problem-Solving Test?

This question was examined on the basis of calculating the mean, variance and standard deviation for the set of scores obtained from the administration of each of the three tests in this study. Table III shows the mean, variance, and standard deviation obtained for each set of test scores from the analysis of the data.

TABLE III

COMPARISON OF THE MEAN, VARIANCE, AND STANDARD DEVIATION FOR THE SETS OF TEST SCORES OBTAINED FROM THE ADMINISTRATION OF THE THREE TESTS

| | Mean Score | Variance | Standard Deviation |
|---|---|---|---|
| Math Vocab. Test | 4.602 | 3.324 | 1.823 |
| Math Concept Test | 7.157 | 3.168 | 1.780 |
| Math Problem-Solving Test | 6.916 | 4.149 | 2.037 |

Note: 10 was the maximum score possible

A mean score of 4.602 was obtained on the Mathematical Vocabulary Test as compared to a mean score of 7.157 on the Concept Test and a mean score of 6.916 on the Problem-Solving Test. The variance and standard deviation on the Mathematical Vocabulary Test were found to be respectively 3.325 and 1.823 as compared to the variance and standard deviation of 3.168 and 1.780 on the Concept Test and 4.149 and 2.037 on the Problem-Solving Test.

CHAPTER V

# SUMMARY, FINDINGS AND CONCLUSION, IMPLICATIONS, AND SUGGESTIONS FOR RESEARCH

## SUMMARY

This study was designed in an attempt to investigate the general hypothesis that there is a significant correlation between an individual's knowledge of mathematical vocabulary and her/his ability to apply mathematical concepts in problem-solving tasks which involve the mathematical vocabulary.

The sample for this study consisted of eighty-three grade ten students enrolled in the mathematics ten program in three high schools in the Edmonton Public School System. One class was selected from each of the three high schools in the above-mentioned system.

The subjects were given three written tests. The first test was a Mathematical Vocabulary Test on which they were required to write definitions for mathematical terms. The second was a Concept Test which involved the vocabulary items from the Mathematical Vocabulary Test and the third was a Problem-Solving Test which also involved the vocabulary items from the Mathematical Vocabulary Test. The

students' performance on the Mathematical Vocabulary Test was correlated with their performance on the Concept Test and with their performance on the Problem-Solving Test. In addition to this, their performance on each of the three tests were compared in order to determine whether the scores would be consistently lower, at the same level, or consistently higher on the Mathematical Vocabulary Test than on the Concept Test or the Problem-Solving Test.

## FINDINGS AND CONCLUSIONS

The findings and conclusions are presented in two sections. In the first section the null hypotheses outlined in Chapter I are restated and the conclusions concerning their acceptance or rejection are discussed. In the second section, the additional question outlined in Chapter I is restated and the findings with respect to this question are discussed.

## NULL HYPOTHESES

### Hypothesis One

There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Concept Test (i.e. a multiple-choice problem-solving test which does not require any computation in

arriving at the correct alternatives).

Analysis of the data revealed that there was a significant correlational relationship, at the .001 level, between the scores on the Mathematical Vocabulary Test and the scores on the Concept Test. Thus, null hypothesis one was rejected.

The analyzed data provided adequate grounds for the rejection of null hypothesis one. A correlational coefficient of $r = .632$ between the performance on the Mathematical Vocabulary Test and the performance on the Concept Test indicates the existence of a 39.9% variance relationship between the variables involved in the knowledge of mathematical concepts that involve this vocabulary. Since problem solving in mathematics requires an understanding of the underlying concepts, a significant relationship between vocabulary knowledge and success in dealing with mathematical concepts indicates a probable extension of this relationship to vocabulary knowledge and problem solving as well.

<u>Hypothesis Two</u>

There is no significant correlational relationship between scores on the Mathematical Vocabulary Test and scores on the Problem-Solving Test (i.e. a mathematical multiple-choice problem-solving test which requires

computation in order to arrive at the correct alternatives).

Analysis of the data revealed that there was a significant correlational relationship, at the .001 level, between the scores on the Mathematical Vocabulary Test and the scores on the Problem-Solving Test. Thus, null hypothesis two was rejected.

The analyzed data provided adequate grounds for the rejection of null hypothesis two. A correlational coefficient of r = .559 between the performance on the Mathematical Vocabulary Test and the performance on the Problem-Solving Test indicates the existence of a 31.2% variance relationship between the variables in the knowledge of mathematical vocabulary and the ability to solve problems that involve this vocabulary.

The results of the analysis of the data with respect to the null hypotheses indicates the existence of a significant relationship between the ability to solve problems in mathematics and the linguistic ability of knowledge of mathematical vocabulary. This is in agreement with the results of research of a similar nature conducted by Eagle (1948), and Johnson (1949).

The findings of this study and those of studies of a similar nature are certainly in support of contentions like those of Stevens (1932). He suggests that failure of students to solve reasoning problems in mathematics is to

some degree caused by deficiencies in reading ability, and claims that it is logical to believe that a student who is unable to understand the situation described in a problem will not be able to solve the problem. The inability to understand the situation in the problem, according to Banks (1959) arises out of the fact that mathematics requires special reading skills and the language of mathematics has characteristics which complicate the reading. These characteristics are due in part to the fact that mathematical vocabulary must convey precise meanings in order to communicate the desired concept to the mind of the reader, and according to Leary (1968) this accounts for one-fourth of the reading difficulties in mathematics. Leary further contends that students will not automatically grow into an understanding and knowledge of this technical vocabulary after they have been introduced to it but must be helped to learn it. Thus it becomes important, according to Barney (1972), that vocabulary development must become an integral part of the teaching of mathematics.

## ADDITIONAL QUESTION

Will the students' scores be consistently lower, at the same level, or consistently higher on the Mathematical Vocabulary Test than on the Concept Test or the Problem-Solving Test?

The mean score on the Mathematical Vocabulary Test of 4.602 was significantly lower than the mean score on the Concept Test of 7.157 or the mean score on the Problem-Solving Test of 6.916. In addition to this, an examination of the individual scores of the students showed that only one student scored lower on the Concept Test than on the Mathematical Vocabulary Test and only six students scored lower on the Problem-Solving Test than on the Mathematical Vocabulary Test. This appears to indicate that students are not able to produce formal definitions to the same level of achievement as in solving problems which involve the mathematical vocabulary they are required to know. This, in turn, seems to indicate that students may have an awareness of the meaning of the mathematical vocabulary to a higher degree than they are able to produce in terms of formal definitions, and as a result can deal with the problems which incorporate this mathematical vocabulary despite the inability to produce satisfactory formal definitions.

This indication is in agreement with Burmeister (1974) who states that we all have two types of vocabularies which can be described as the receptive and the expressive. Our receptive vocabulary is made up of words we recognize when we read and/or listen and is several times larger than our expressive vocabulary which is made

up of words we use when we speak and/or write. An examination of the implication of this dichotomy leads one to conclude that our ability to produce formal definitions for words is likely even smaller than our expressive vocabulary.

## IMPLICATIONS

In view of the findings of this study the following implications would appear to be of importance to those involved in the teaching of mathematics.

1. Teachers of mathematics need to be made aware of the relationship of the knowledge of mathematical vocabulary to success in problem solving in mathematics.

2. Students need to be given specific classroom instruction, in mathematics, in reading techniques with respect to the learning of mathematical vocabulary as well as any other new vocabulary they may encounter in the process of learning mathematical concepts.

3. Teachers of mathematics can assist their students in gaining greater proficiency in problem solving by selecting those problems which employ vocabulary close to the students' reading level in terms of familiar vocabulary.

4. Teachers of mathematics need to be made aware of and periodically reminded that they must assume the responsibility of teaching reading and reading skills .

whenever and wherever they are needed. That is to say the identification of specific reading skills required by the students and determination of procedures and techniques necessary to teach those skills must become the responsibility of each teacher of mathematics. To do this, however, teachers of mathematics should seek assistance from specialists in reading.

5. The individuals responsible for the selection of mathematics text books should be made aware of the relationship between the reading aspect of vocabulary knowledge and students' success in problem solving in order that they make text book selections as close to the student's grade level as possible.

## SUGGESTIONS FOR RESEARCH

As a result of the findings of this study, there appears to be a need for the following kinds of research.

1. There is a need for the results of this study to be verified with a larger random sample of subjects at various grade levels and with different mathematical content.

2. There is a need for research to be conducted on the various levels of knowledge of specialized mathematical vocabulary and the relationship of these levels to success to problem solving in Mathematics.

3. There is a need for research at various grade levels and on various mathematical content on the relationship of students' ability to cope with syntactic structure to their success in problem solving in mathematics.

4. There is a need for research to determine whether the teaching of reading in mathematics will increase success in problem solving.

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APPENDIX A

MATHEMATICAL VOCABULARY TEST

# TEST A

Write the definition for each of the following mathematical terms in the space provided.

1. Reflex angle- ______________________________

2. Alternate angles- ______________________________

3. Isosceles triangle- ______________________________

4. Straight angle- ______________________________

5. Deductive reasoning- ______________________________

6. Vertical angles- ______________________________

________________________________________

________________________________________

________________________________________

7. Perpendicular bisector- ______________________

________________________________________

________________________________________

________________________________________

8. Median of a triangle- ________________________

________________________________________

________________________________________

________________________________________

9. Right triangle- ______________________________

________________________________________

________________________________________

________________________________________

10. Complementary angles- ________________________

________________________________________

________________________________________

________________________________________

# APPENDIX B

# CONCEPT TEST

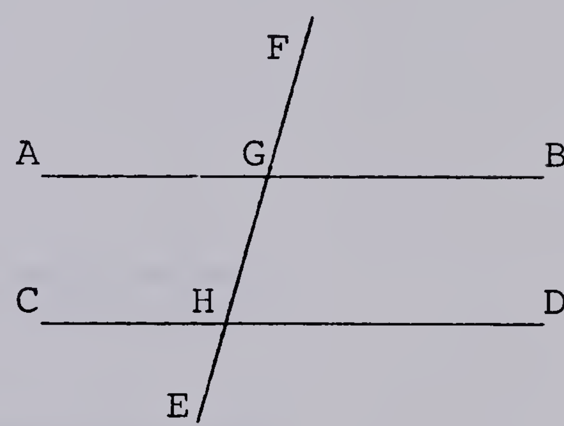
## TEST B

Select the answer for each of the following questions from the given alternatives.

1. Choose the answer which arranges the following angles in decreasing order of size.

    a. obtuse, right, reflex
    b. reflex, obtuse, right
    c. right, obtuse, acute
    d. right, reflex, obtuse

2. In the given diagram, $\angle AGH$ and $\angle DHG$ are what kind of angles?

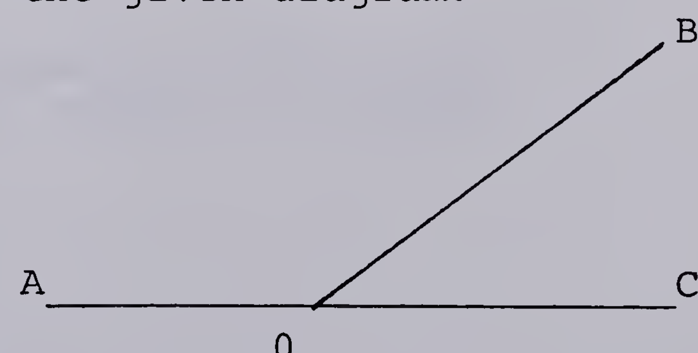



    a. alternate angles
    b. corresponding angles
    c. exterior angles on the same side of the transversal
    d. interior angles on the same side of the transversal

3. If two angles of a triangle are equal, one can conclude that the triangle is:

    a. acute angled
    b. equilateral
    c. isosceles
    d. scalene

4. One of the postulates of geometry reads "if two adjacent angles together form two right angles, their exterior arms are in the same straight line." According to this postulate which of the following assertions is false in regard to the given diagram?



AOC is a straight line if:

a. AO and OC are straight line segments
b. $\angle$ COB + $\angle$ BOA = 180 degrees
c. $\angle$ COB + $\angle$ BOA is a straight line
d. $\angle$ COB and $\angle$ BOA are supplementary

5. GENERAL STATEMENT: If equals are divided by equals, the quotients are equal.
SPECIFIC STATEMENT: $6x = 12$
CONCLUSION: $x = 2$
This is an example of:

a. inductive reasoning
b. analytic reasoning
c. generalization
d. deductive reasoning

6. "If two angles are vertically opposite each other, they are equal." Using only the above information, which of the following is true?

a. If angles are not vertically opposite each other, they are not equal.
b. If two angles are equal, they are vertically opposite each other.
c. Two unequal angles cannot be vertically opposite each other.
d. Equal angles are vertically opposite angles.

7. The line formed by the set of all points equidistant from the end of a given line is called a:

a. median
b. perpendicular bisector
c. midray
d. transversal

8. A line joining any vertex of a triangle to the middle point of the opposite side of the triangle is called a (n):

a. angle bisector
b. median
c. right bisector
d. altitude

9. Right triangles can never be:

    a. equilateral
    b. isosceles
    c. scalene
    d. congruent

10. In triangle PQR, angles P and Q are complementary. The triangle must be:

    a. equilateral
    b. isosceles
    c. obtuse angled
    d. right angled

APPENDIX C

PROBLEM-SOLVING TEST

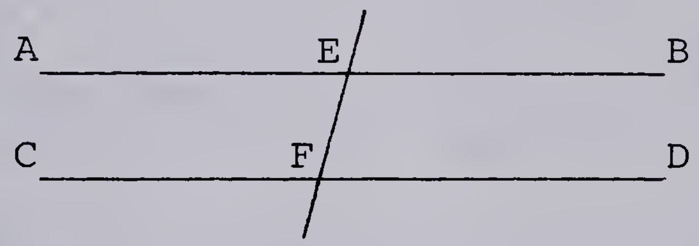
# TEST C

Choose the correct solution to each of the following problems from the given alternatives.

1. If a reflex angle has a measure of $x^{o}$, then the possible values of x are:

   a. $180 < x < 360$
   b. $90 < x < 360$
   c. $90 < x < 180$
   d. $0 < x < 90$

2.



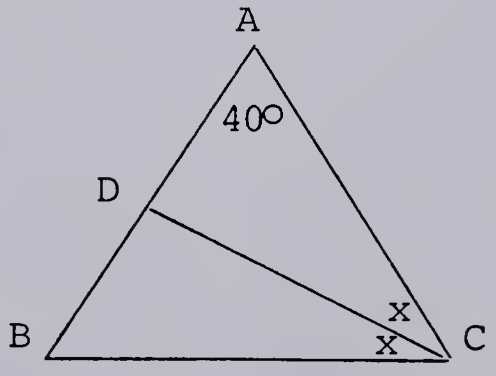

$\angle AEF = (2x - 40)$ degrees
$\angle DFE = (x + 20)$ degrees

If AB is parallel to CD, then $\angle AEF$ is:

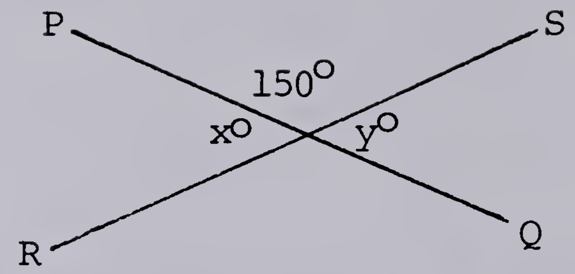
   a. 20 degrees
   b. 40 degrees
   c. 60 degrees
   d. 80 degrees

3. In the isosceles triangle ABC, AB = AC and DC bisects $\angle ACB$. The measure of x in degrees is:

   a. 20
   b. 70
   c. 40
   d. 35

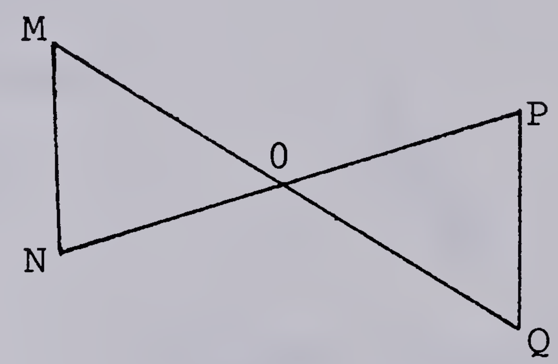


4. In the diagram, PQ and RS are intersecting lines. The value of $(x + y)^{o}$ is:




a. 15
b. 30
c. 60
d. 90

5. In the diagram, MQ and NP are bisectors of each other. If asked to prove MN ≅ QP, the statements, without authorities of the best proof would be:



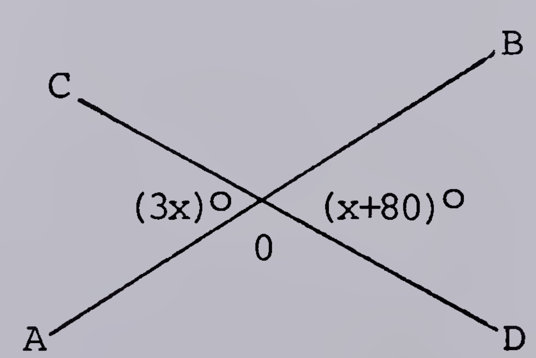

a. MN ≅ QP
MO ≅ QO
NO ≅ PO
△ MNO ≅ △ QPO (SSS)
MN ≅ QP

b. MO ≅ QO
∠ MNO ≅ ∠ PQO
MN ≅ QP
△ MNO ≅ △ QPO (SAS)
MN ≅ QP

c. NO ≅ PO
MO ≅ QO
∠ NOM ≅ ∠ POQ
△ MNO ≅ △ QPO (SAS)
MN ≅ QP

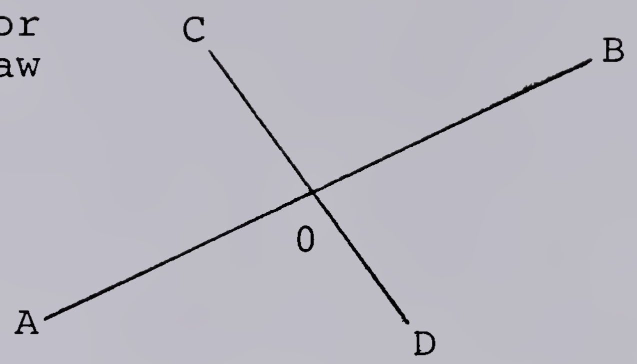
d. ∠MNO ≅ ∠ QPO
MN ≅ QP
NO ≅ PO
△ MNO ≅ △ QPO (SAS)
MN ≅ PQ

6. In the diagram, lines AB and CD intersect at 0. How many degrees are there in ∠ COA?




a. 20
b. 40
c. 60
d. 120

7. CD is the perpendicular bisector of AB. A conclusion we can draw from this is:

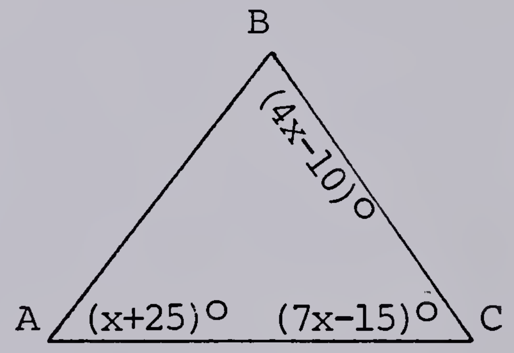



   a. CA = DA
   b. CB = BD
   c. CA = CB
   d. CO = DO

8. In triangle ABC, the median is the altitude. We can conclude that:

   a. triangle ABC is scalene
   b. triangle ABC is isosceles
   c. triangle ABC is oblique
   d. triangle ABC is right angled

9. In the diagram, triangle ABC is:




   a. equilateral
   b. obtuse angled
   c. right angled
   d. congruent angled

10. If an angle is nine times its complement, the angle is:

   a. 8 degrees
   b. 81 degrees
   c. 72 degrees
   d. 9 degrees